

# Effective operators in SUSY, superfield constraints and searches for a UV completion

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## Abstract

We discuss the role of a class of higher dimensional operators in 4D N=1 supersymmetric effective theories. The Lagrangian in such theories is an expansion in momenta below the scale of “new physics” ( $\Lambda$ ) and contains the effective operators generated by integrating out the “heavy states” above  $\Lambda$  present in the UV complete theory. We go beyond the “traditional” leading order in this momentum expansion (in  $\partial/\Lambda$ ). Keeping manifest supersymmetry and using superfield *constraints* we show that the corresponding higher dimensional (derivative) operators in the sectors of chiral, linear and vector superfields of a Lagrangian can be “unfolded” into second-order operators. The “unfolded” formulation has only polynomial interactions and additional massive superfields, some of which are ghost-like if the effective operators were *quadratic* in fields. Using this formulation, the UV theory emerges naturally and fixes the (otherwise unknown) coefficient and sign of the initial (higher derivative) operators. Integrating the massive fields of the “unfolded” formulation generates an effective theory with only polynomial effective interactions relevant for phenomenology. We also provide several examples of “unfolding” of theories with higher derivative *interactions* in the gauge or matter sectors that are actually ghost-free. We then illustrate how our method can be applied even when including *all orders* in the momentum expansion, by using an infinite set of superfield constraints and an iterative procedure, with similar results.

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## 1 Introduction

Effective field theories are our main tool for studying physics at high scales, like the physics beyond the Standard Model. There are two reasons for this. One reason is the absence of a fundamental theory (UV completion). The second reason is that these theories are convenient for practical purposes: we do not need them if we have the full theory and are able to compute everything in such case; but effective theories make calculations much easier by focusing on the *relevant parameters* for the physics at the (momentum) scale investigated. Then shorter distance physics can be ignored together with all particles too heavy to be produced at this scale. Eliminating (*i.e.* integrating out) these particles simplifies the calculation. The result is a non-renormalizable theory (even if initial theory was renormalizable), in which all nontrivial effects of heavy particles appear in operators with dimensions  $d > 4$  [1, 2]. In the full theory, these effects are included in the non-local interactions obtained by integrating out

“heavy particles”. But in the effective theories one replaces non-local interactions with virtual particles exchange by a set of *local* interactions such as to give the same low energy physics. The high energy behaviour is affected, so the effective theory is only valid at momenta below the mass of the “heavy particles”. This mass is the effective theory cut-off ( $\Lambda$ ).

The Lagrangian of the effective theory then contains just local interactions, obtained from an expansion in momenta below this scale, i.e. in powers of  $\partial/\Lambda$ , up to some finite order. Keeping *all orders* in momenta leads to a *non-local* theory equivalent to the original full theory<sup>1</sup>. The effective Lagrangian is analytic in  $\partial/\Lambda$  in the region relevant to the low energy theory and it can be dealt with in any finite order in the momentum expansion. This is a local Lagrangian that one is using. This picture is so familiar that it is usually implicitly assumed, but we reminded it to make clear our set up. For a review see [2].

In this work we would like to investigate such effective theories beyond the first leading term of the momentum expansion. This is relevant when the momentum is closer to  $\Lambda$ . Using this picture we also try to infer a UV theory at scales above the “heavy particles” mass. By “UV theory” we mean a two-derivative ghost-free theory with only polynomial interactions in superfields. If this theory is renormalizable we refer to it as a UV completion<sup>2</sup>.

Let us formulate the above picture and our goals in a more precise way. In 4D SUSY effective theories the study is often strongly restricted to Kahler potentials  $K$  and superpotentials  $W$  that depend only on the superfields  $\Phi^i$ . But one often encounters cases when  $K$  and  $W$  depend on more general arguments such as the superderivatives  $D_\alpha$  acting on the superfields i.e.  $K = K(\Phi^i, \Phi_j^\dagger, \overline{D}\Phi_k^\dagger, D\Phi^l, \overline{D}^2\Phi_m^\dagger, D^2\Phi^n, \dots)$  and  $W = W(\Phi^i, \overline{D}^2\Phi_i^\dagger, \dots)$ . In component fields, this action contains powers of  $\partial^\mu$ ,  $\square$ , etc. These account for the momentum expansion  $\partial/\Lambda$  giving the effective theory mentioned.  $\Lambda$  is here related to Kahler curvature.

For the two-point Green functions (propagators) the presence of an expansion in powers of  $\partial/\Lambda$  (giving higher dimensional derivative operators) can lead to additional poles; some of these are ghosts, (super)fields of negative kinetic terms. There is nothing “pathological” about their presence here. They are just artefacts of the effective approach that are eliminated by the field equations or non-linear field redefinitions, to leave a ghost-free effective theory. As mentioned, the corresponding (derivative) operators are a common presence in the low energy limit of the UV theory, after integrating out the “heavy particles”. They are thus related to the UV completion of the effective action. Such operators are also present in the interactions terms. When scalar fields in these interactions develop vev’s, these terms can contribute to the two-point Green functions, with similar effects (additional poles and ghosts). These operators are studied below.

Let us detail how such operators emerge when classically integrating out massive states.

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<sup>1</sup>There is nothing wrong in using this non-local theory for calculations [2]. For the differences between this non-local and effective local theories, see [2]; we shall meet such an example in Section 5 and the Appendix.

<sup>2</sup>An example of a UV theory is 4D N=1 supergravity (UV incomplete).

Consider a simple (UV complete) Lagrangian

$$\mathcal{L} = \int d^4\theta \left\{ \Phi_0^\dagger \Phi_0 + \chi^\dagger \chi \right\} + \left\{ \int d^2\theta \left[ (1/8) \Lambda \chi^2 + (1/4) m_0 \Phi_0 \chi + W(\Phi_0) \right] + h.c. \right\} \quad (1)$$

For large  $\Lambda$ , the superfield  $\chi$  has a large mass, so it can be integrated out using its equation of motion  $-\bar{D}^2 \chi^\dagger + \Lambda \chi + m \Phi_0 = 0$ . This has an iterative solution

$$\chi = \frac{-m_0}{\Lambda} \Phi_0 + \frac{-m_0}{\Lambda^2} \bar{D}^2 \Phi_0^\dagger + \frac{-m_0}{\Lambda^3} \bar{D}^2 D^2 \Phi_0 + \dots \quad (2)$$

Thus  $\chi$  is an infinite series in  $(\partial/\Lambda)$ . This solution is used back in  $\mathcal{L}$  to give

$$\mathcal{L}_{\text{eff}} = \int d^4\theta \Phi_0^\dagger \left[ 1 - \frac{16m_0^2}{\Lambda^4} \square \right] \Phi_0 + \left\{ \int d^2\theta \left[ W(Z^{\frac{1}{2}} \Phi_0) + \frac{2m_0^2}{\Lambda^3} \Phi_0 \square \Phi_0 \right] + h.c. \right\} + \dots \quad (3)$$

where we replaced  $\bar{D}^2 D^2 \rightarrow -16\square$  and  $Z = 1/(1 + m^2/\Lambda^2)$ . So a simple decoupling of a massive state generated the  $\square$ -operators. Actually, due to eq.(2), eq.(3) contains an *infinite* series in momentum expansion  $(\partial/\Lambda)$ , from the initial renormalizable theory. One usually truncates this series to a low(est) order, as in (3). Then  $\square$ -operators are often eliminated by simply using the leading order equations of motion<sup>3</sup> (for the non-SUSY case, see [3, 4]).

Such operators are also generated dynamically by compactification, as loop counterterms. They can be generated by bulk (gauge) interactions to give the F-term below that contributes to a one-loop running of the 4D effective gauge coupling in orbifolds [5, 6, 7, 8, 9, 10, 11]

$$\delta\mathcal{L} \supset R^2 \int d^4\theta \Phi^\dagger \square \Phi + \left\{ R^2 \int d^2\theta W^\alpha \square W_\alpha + h.c. \right\} \quad (4)$$

where  $1/R$  is the compactification scale and  $W^\alpha$  is the gauge field strength. Regarding the D-term, it is generated by superpotential (Yukawa) interactions localised at the 4D fixed points of an orbifold (it can be a Higgs mass counterterm in such orbifolds [12, 13]). So models with extra dimensions contain such effective operators<sup>4</sup> at one-loop. Such operators can also be present in other compactifications (Randal-Sundrum, etc). In conclusion, these operators are common and are related to the UV regime.

The main goal of this work is to clarify two problems for general effective theories:

**1):** to obtain a better understanding of the higher order terms in momentum expansion, and then “remove” these higher derivative operators from the effective Lagrangian. To do this we show that one can reformulate (“unfold”) such a Lagrangian into a *second-order* Lagrangian

<sup>3</sup>In SUSY theories with higher derivative terms, the auxiliary fields become dynamical in most cases.

<sup>4</sup> The relation of these results to string theory is discussed in [6, 10, 11].

i.e. without higher dimensional (derivative) operators. This is what we mean by “unfolding”. This result is interesting since a two-derivative formulation of a theory is easier to handle and the “unfolded” formulation is a first step towards identifying a (ghost-free) UV theory.

**2):** to identify a two-derivative ghost-free UV theory of the effective theory with such operators. It is of strong interest to find a UV quantum consistent theory leading in the infrared to these (effective) operators and to fix in this way their (otherwise unknown) coefficient and sign, in agreement with constraints derived from analyticity and causality [14]. The “unfolded” formulation will help us to achieve this. For a related discussion see [15, 16, 17, 18].

We show how to “remove” the  $\square$ -operators acting on chiral, vector or linear superfields, in a consistent way, while preserving manifest supersymmetry (in a superfield language). We show how (effective) theories that contain such operators can be “unfolded” into second order theories with only polynomial interactions and with additional *massive* states, sometimes with negative kinetic terms (ghosts). For this, one eliminates these operators by suitable superfield *constraints*. For example, in the chiral sector such constraints replace each  $\overline{D}^2\Phi^\dagger$  by a superfield  $m\Phi' = \overline{D}^2\Phi^\dagger$ . Here  $m$  is a small *arbitrary* scale of the theory that enforces the constraint. All superderivatives are thus eliminated, to find a second-order theory. The method can be iterated to higher orders in  $D_\alpha, \overline{D}^{\dot{\alpha}}$  and can also be applied to non-derivative effective operators. Subsequent integration of these massive states leads to a theory with effective *polynomial* operators only and this formulation can be used for phenomenology.

We then show how the “unfolded” formulation helps us identify a UV theory of the initial effective theory. In the above models the initial effective operator was quadratic in fields. We also study other models with higher derivative interactions that are *ghost-free*: a chiral superfield model with such operators, a model of Dirac gaugino masses and a supersymmetric version of the Euler-Heisenberg Lagrangian. We “unfold” them and then find their UV theory.

The plan of the paper is as follows. For the operators of eqs.(3), (4) acting on chiral, linear and gauge superfields the “unfolding” method is done in Sections 2, 3 and 4. This extends our study in [15]. For related discussions see [19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30]. In each of these Sections we use the “unfolded” formulation to identify a UV theory (or UV completion) that generates at low energy the effective operators considered. Section 2 also contains the unfolding of a higher-derivative ghost-free chiral superfield model. Section 4 contains ghost-free examples leading to Dirac gaugino masses and to the Euler-Heisenberg gauge Lagrangian. Section 5 comments on how to treat more general cases. The Appendix presents the “unfolding” of a theory with an (known) infinite series of superderivatives. Using *an infinite* set of superfield constraints it is shown that even in this case there exists an “unfolded” version with only polynomial (d=4) interactions and an infinite set of extra (massive) superfields. Truncating the “unfolded” theory to a number of such fields is equivalent to the truncation to a corresponding power in  $(\partial/\Lambda)$  of the momentum expansion.

## 2 Effective operators in the chiral sector

### 2.1 “Unfolding” the effective operators

Let us first consider the case the  $\square$ -operators, eqs.(3), (4) in the matter sector [15]. Consider

$$\mathcal{L} = \int d^4\theta \left[ \Phi_1^\dagger \Phi_1 + \frac{\rho}{\Lambda^2} \Phi_1^\dagger \square \Phi_1 \right] + \left\{ \int d^2\theta \left[ \frac{\sigma}{\Lambda} \Phi_1 \square \Phi_1 + W(\Phi_1) \right] + \text{h.c.} \right\} + \mathcal{O}(1/\Lambda^3) \quad (5)$$

where  $\rho, \sigma = \mathcal{O}(1)$  are independent. We replace  $\square$  by  $(-1/16)\overline{D}^2 D^2$ . Further, introduce  $\Phi_2$

$$\overline{D}^2 \Phi_1^\dagger - m \Phi_2 = 0. \quad (6)$$

where  $m$  is a real, small but arbitrary mass scale of the theory. This is a superfield constraint that we add to  $\mathcal{L}$ , using a Lagrange multiplier superfield<sup>5</sup>  $\Phi_3$ . Then

$$\begin{aligned} \mathcal{L} = & \int d^4\theta \left[ \Phi_1^\dagger \Phi_1 - \frac{\rho m^2}{16\Lambda^2} \Phi_2^\dagger \Phi_2 \right] \\ & + \left\{ \int d^2\theta \left[ W(\Phi_1) - \frac{\sigma m}{16\Lambda} \Phi_1 \overline{D}^2 \Phi_2^\dagger - \frac{m}{4\Lambda} \Phi_3 (\overline{D}^2 \Phi_1^\dagger - m \Phi_2) \right] + \text{h.c.} \right\} + \mathcal{O}(1/\Lambda^3) \end{aligned} \quad (7)$$

The field equation for  $\Phi_3$  recovers the constraint. Also, this constraint is implemented with a coefficient  $1/\Lambda$  because it must be removed when  $\Lambda \rightarrow \infty$ , while  $m$  is an unphysical parameter that restores the mass dimension (at the end of the calculation we take  $m \rightarrow 0$ ).  $\mathcal{L}$  becomes

$$\begin{aligned} \mathcal{L} = & \int d^4\theta \left[ \Phi_1^\dagger \Phi_1 - \frac{\rho m^2}{16\Lambda^2} \Phi_2^\dagger \Phi_2 + \frac{m}{4\Lambda} (\sigma \Phi_1^\dagger \Phi_2 + \sigma^* \Phi_1 \Phi_2^\dagger) + \frac{m}{\Lambda} (\Phi_1^\dagger \Phi_3 + \Phi_1 \Phi_3^\dagger) \right] \\ & + \left\{ \int d^2\theta \left[ W(\Phi_1) + \frac{m^2}{4\Lambda} \Phi_2 \Phi_3 \right] + \text{h.c.} \right\} + \mathcal{O}(1/\Lambda^3) \end{aligned} \quad (8)$$

The (hermitian) matrix  $k_{ij}$  of the kinetic (D-)terms  $\Phi_i^\dagger k_{ij} \Phi_j$  has  $\det k_{ij} = \rho m^4/(16\Lambda^4)$ . Its real eigenvalues<sup>6</sup> control the nature of the superfields  $\Phi_{1,2,3}$ : positive (negative) eigenvalues correspond to particle-like (ghost-like) superfields, respectively. We therefore have:

a) If  $\rho = 1$  then we have two negative and one positive eigenvalue, the latter corresponding to the original particle-like degree of freedom. Two superghosts are present, one due to the operator  $\square$ , the second because auxiliary  $F_1$  of  $\Phi_1$  became dynamical, thus an extra d.o.f. is present which, by supersymmetry, demands the presence of an extra (ghost) superfield.

<sup>5</sup>This Lagrange multiplier method is similar to the one used in [31] in a case without higher derivatives.

<sup>6</sup>The matrix is  $k_{11} = 1$ ,  $k_{12} = \sigma m/(4\Lambda) = k_{21}^*$ ,  $k_{13} = m/\Lambda = k_{31}^*$ ,  $k_{22} = -\rho m^2/(16\Lambda)$ ,  $k_{23} = 0 = k_{32} = k_{33}$ .

- b) If  $\rho = -1$ , one has one negative (1 ghost) and two positive (2 particles) eigenvalues<sup>7</sup>.  
c) If  $\rho = 0$  then one eigenvalue is 0, one is positive and one is negative. Thus we have one ghost and one particle superfields. All these eigenvalues are the roots of

$$-\nu^3 + [1 - \rho m^2/(16\Lambda^2)] \nu^2 + (1 + \rho/16 + |\sigma|^2/16) \nu m^2/\Lambda^2 + \rho m^4/(16\Lambda^4) = 0 \quad (9)$$

The exact expressions can be obtained. According to our discussion we have

$$\nu_1 > 0, \quad \nu_2 < 0, \quad \nu_3 \sim -\rho \quad (10)$$

The notation  $\nu_3 \sim -\rho$  means  $\nu_3$  has the sign of  $(-\rho)$  and is 0 if  $\rho = 0$ . This covers all cases discussed above. For a complete analysis, we also bring  $\mathcal{L}$  to canonical form using a transformation to  $\Phi'_i = u_{ij} \Phi_j$  with  $\text{diag}(\nu_1, \nu_2, \nu_3) = u k u^\dagger$  and  $u_{ij}$  unitary. With the notation  $z_{kj} = u_{k2}^* u_{j3}^* m^2/(4\Lambda)$  and after rescaling  $\tilde{\Phi}'_i = \tilde{\Phi}_i/\sqrt{|\nu_i|}$ , ( $\nu_i \neq 0$ ), one finds

$$\mathcal{L} = \int d^4\theta [\tilde{\Phi}_1^\dagger \tilde{\Phi}_1 - \tilde{\Phi}_2^\dagger \tilde{\Phi}_2 - \rho \tilde{\Phi}_3^\dagger \tilde{\Phi}_3] + \left\{ \int d^2\theta \left[ W \left( \frac{u_{j1}^* \tilde{\Phi}_j}{\sqrt{|\nu_j|}} \right) + \frac{z_{ij} \tilde{\Phi}_i \tilde{\Phi}_j}{\sqrt{|\nu_i|} \sqrt{|\nu_j|}} \right] + \text{h.c.} \right\} + \mathcal{O}(1/\Lambda^3) \quad (11)$$

The initial  $\mathcal{L}$  was “unfolded” into a second order theory, with extra superfields  $\tilde{\Phi}_{2,3}$  (of mass  $\sim \Lambda$ , see later) and at least one of them having “wrong”-sign kinetic term (superghost). None of the auxiliary fields is dynamical anymore. The effective operators are not present anymore and all interactions are polynomial, up to  $\mathcal{O}(1/\Lambda^3)$ .

If  $\sigma = 0$  then, with  $\rho = \pm 1$ :

$$\mathcal{L} = \int d^4\theta [\tilde{\Phi}_1^\dagger \tilde{\Phi}_1 - \tilde{\Phi}_2^\dagger \tilde{\Phi}_2 - \rho \tilde{\Phi}_3^\dagger \tilde{\Phi}_3] + \left\{ \int d^2\theta [W(\tilde{\Phi}_1 - \tilde{\Phi}_2) + \Lambda \tilde{\Phi}_2 \tilde{\Phi}_3] + \text{h.c.} \right\} + \mathcal{O}(1/\Lambda^3) \quad (12)$$

If instead  $\rho = 0$  and  $\sigma = \pm 1$

$$\mathcal{L} = \int d^4\theta [\tilde{\Phi}_1^\dagger \tilde{\Phi}_1 - \tilde{\Phi}_2^\dagger \tilde{\Phi}_2] + \left\{ \int d^2\theta [W(\tilde{\Phi}_1 - \tilde{\Phi}_2) + (1/4) \sigma \Lambda \tilde{\Phi}_2^2] + \text{h.c.} \right\} + \mathcal{O}(1/\Lambda^3) \quad (13)$$

Similar expressions exist for the more general case when  $\rho \neq 0$  and  $\sigma \neq 0$  simultaneously. Note that in the arguments of  $W$  in the last two equations and also inside the square bracket of the F-terms there are additional terms  $\mathcal{O}(m/\Lambda)$  that we did not write since we now set  $m \rightarrow 0$  because we do not have a constraint anymore. As a side remark, notice that the scalar potential is  $V = |\tilde{F}_1|^2 - |\tilde{F}_2|^2 - \rho |\tilde{F}_3|^2$  in eq.(12) and  $V = |\tilde{F}_1|^2 - |\tilde{F}_2|^2$  in eq.(13) where  $\tilde{F}_i$  are the auxiliaries of superfields  $\tilde{\Phi}_i$ . This allows  $V = 0$  with broken *global* SUSY.

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<sup>7</sup>We cannot have 3 negative eigenvalues, since we had one positive value to begin with (the initial particle)

The effect of the original higher dimensional operators was then to introduce ghost superfields, of large mass (of order  $\Lambda$ ) as shown by the F-terms in the last two equations<sup>8</sup>. Using the equations of motion one can integrate out the massive ghost superfields. For example in eq.(13), one uses the equation of motion for  $\Phi_2$

$$(-1/4)\overline{D}^2\tilde{\Phi}_2 - W'(\tilde{\Phi}_1 - \tilde{\Phi}_2) + (1/2)\sigma\Lambda\tilde{\Phi}_2 = \mathcal{O}(1/\Lambda^3) \quad (14)$$

where the derivative of  $W$  is wrt its shown argument. This gives

$$\tilde{\Phi}_2 = \frac{2\sigma}{\Lambda} W'(\tilde{\Phi}_1 - \tilde{\Phi}_2) - \frac{1}{\Lambda^2}\overline{D}^2 W'(\tilde{\Phi}_1 - \tilde{\Phi}_2) + \mathcal{O}(1/\Lambda^3) \quad (15)$$

One then expands  $W'(\tilde{\Phi}_1 - \tilde{\Phi}_2) = W' - \tilde{\Phi}_2 W'' + (1/2)\tilde{\Phi}_2^2 W''' + \mathcal{O}(1/\Lambda^3)$  where we introduced the notation  $W' \equiv W'(\tilde{\Phi}_1)$   $W'' \equiv W''(\tilde{\Phi}_1)$ . Therefore

$$\tilde{\Phi}_2 = \frac{2\sigma}{\Lambda} W' - \frac{1}{\Lambda^2}\overline{D}^2 W' - \frac{4}{\Lambda^2} W' W'' + \mathcal{O}(1/\Lambda^3) \quad (16)$$

Using this and expanding  $W(\tilde{\Phi}_1 - \tilde{\Phi}_2)$  about  $\tilde{\Phi}_1$  in eq.(13), one obtains  $\mathcal{L}_{\text{eff}}$

$$\mathcal{L}_{\text{eff}} = \int d^4\theta \left[ \tilde{\Phi}_1^\dagger \tilde{\Phi}_1 - \frac{4}{\Lambda^2} |W'|^2 \right] + \left\{ \int d^2\theta \left[ W - \frac{\sigma}{\Lambda} W'^2 + \frac{2}{\Lambda^2} W'^2 W'' \right] + \text{h.c.} \right\} + \mathcal{O}(1/\Lambda^3) \quad (17)$$

where  $W \equiv W(\tilde{\Phi}_1)$  and  $\sigma = \pm 1$ . Thus effective operators re-emerge but are now *polynomial* in fields.  $\mathcal{L}_{\text{eff}}$  is classically equivalent to starting  $\mathcal{L}$  in eq.(5) for  $\rho = 0$ . Even if  $W'$  may contain a linear dependence on  $\tilde{\Phi}_1$ , no ghost can be generated in the D-term due to the suppression  $1/\Lambda^2$  (relative to dominant  $\tilde{\Phi}_1^\dagger \tilde{\Phi}_1$ ). Eq.(17) can now be used for phenomenology.

For eq.(12) integrating the superghosts  $\tilde{\Phi}_2, \tilde{\Phi}_3$  of mass  $\sim \Lambda$  is done similarly to find

$$\mathcal{L}_{\text{eff}} = \int d^4\theta \left[ \tilde{\Phi}_1^\dagger \tilde{\Phi}_1 - \frac{\rho}{\Lambda^2} |W'(\tilde{\Phi}_1)|^2 \right] + \left\{ \int d^2\theta W(\tilde{\Phi}_1) + \text{h.c.} \right\} + \mathcal{O}(1/\Lambda^3) \quad (18)$$

which is equivalent to eq.(5) for  $\sigma = 0$ . This is the formulation that can be used for phenomenology. In both examples there are no ghost superfields as asymptotic (final) states in the approximation  $\mathcal{O}(1/\Lambda^3)$ . A similar result is obtained if both  $\rho, \sigma \neq 0$ .

The method can be extended to more general cases and higher orders, etc. An alternative to our approach that leads to results identical to those in eqs.(17), (18), is to use in eq.(5) non-linear field redefinitions to “remove” the derivative operators [18].

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<sup>8</sup> The difference in the number of superghosts is because if  $\rho \neq 0$  the auxiliary field of  $\Phi$  becomes dynamical (unlike the case of  $\rho = 0$ ) and by SUSY this brings an extra superfield in the “unfolded” Lagrangian.

## 2.2 A ghost-free UV theory in the matter sector

The above result indicates how a UV theory of the starting Lagrangian eq.(5) can be realised, that is ghost free and, in this case, also renormalizable (UV complete). If we ignore the form of the dimensionless constants and  $\mathcal{O}(1/\Lambda^3)$  terms, Lagrangian (8) contains only dimension-four operators. The only problem is that it has ghosts, so it is not UV complete. This is seen after its kinetic terms are diagonalised, leading to results (11) to (13) in which ghost superfields emerge. Their presence is induced by the kinetic mixing in (8) that is dominant in the D-term, due to the absence of diagonal kinetic terms for  $\Phi_2$  and  $\Phi_3$ . This indicates that a ghost free theory should thus “UV complete” Lagrangian (8) by the addition of diagonal kinetic terms

$$\delta\mathcal{L} = \zeta \Phi_2^\dagger \Phi_2 + \eta \Phi_3^\dagger \Phi_3 \quad (19)$$

with suitable values for real  $\zeta, \eta$ . Let us examine the impact of these terms. The new  $\mathcal{L}$  becomes

$$\begin{aligned} \mathcal{L} = & \int d^4\theta \left[ \Phi_1^\dagger \Phi_1 + \left( \zeta - \frac{\rho \xi^2}{16} \right) \Phi_2^\dagger \Phi_2 + \frac{\xi}{4} (\sigma \Phi_1^\dagger \Phi_2 + \sigma^* \Phi_1 \Phi_2^\dagger) + \xi (\Phi_1^\dagger \Phi_3 + \Phi_1 \Phi_3^\dagger) + \eta \Phi_3^\dagger \Phi_3 \right] \\ & + \left\{ \int d^2\theta \left[ W(\Phi_1) + \frac{1}{4} m \xi \Phi_2 \Phi_3 \right] + \text{h.c.} \right\} \end{aligned} \quad (20)$$

with  $\xi$  real<sup>9</sup>. Let us first show that this UV completed theory, recovers at low energy the Lagrangian in eq.(5). The equations of motion for  $\Phi_{2,3}$  are

$$\begin{aligned} \Phi_3 &= \frac{\sigma}{4m} \overline{D}^2 \Phi_1^\dagger + \frac{1}{m^2 \xi} \left( \zeta - \frac{\rho \xi^2}{16} \right) \overline{D}^2 D^2 \Phi_1 + \mathcal{O}(1/m^3) \\ \Phi_2 &= \frac{1}{m} \overline{D}^2 \Phi_1^\dagger + \frac{\eta \sigma^*}{4m^2 \xi} \overline{D}^2 D^2 \Phi_1 + \mathcal{O}(1/m^3) \end{aligned} \quad (21)$$

Using this back in  $\mathcal{L}$  gives, up to  $\mathcal{O}(1/m^3)$  terms

$$\mathcal{L} = \int d^4\theta \left[ \Phi_1^\dagger \Phi_1 + \frac{16}{m^2} \left( \frac{\rho \xi^2}{16} - \zeta - \frac{\eta |\sigma|^2}{16} \right) \Phi_1^\dagger \square \Phi_1 \right] + \left\{ \int d^2\theta \left[ W(\Phi_1) + \sigma^* \xi \Phi_1 \square \Phi_1 \right] + \text{h.c.} \right\} \quad (22)$$

This  $\mathcal{L}$  is identical to the original Lagrangian of eq.(5) provided that

$$\zeta = -\eta |\sigma|^2 / 16 \quad (23)$$

We thus need only one extra parameter  $\eta$  to find a UV theory.

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<sup>9</sup>  $\xi$  is just a dimensionless parameter ( $\xi \rightarrow m/\Lambda$  in Section 2). In the UV theory  $\xi$  ( $m$ ) becomes physical.

As a UV theory, this theory must be ghost-free, so let us check under what conditions this is true. This happens if we have positive values for the eigenvalues  $\tilde{\nu}_{1,2,3}$  of the matrix of the quadratic form of the D-term in eqs.(20), (23);  $\tilde{\nu}_{1,2,3}$  are the roots of

$$\begin{aligned} -\tilde{\nu}^3 &+ \tilde{\nu}^2 (\eta + \zeta + 1 - \rho \xi^2/16) + \tilde{\nu} \left[ \xi^2 (1 + (\rho + |\sigma|^2 + \eta\rho)/16) - \zeta - \eta - \eta \zeta \right] \\ &+ (\zeta - \rho \xi^2/16) (\eta - \xi^2) - \eta \xi^2 |\sigma|^2/16 = 0 \end{aligned} \quad (24)$$

with  $\zeta$  as in eq.(23). Let us determine the values of the new coefficient  $\eta$  so that  $\tilde{\nu}_{1,2,3} \geq 0$ .

- a). if  $\sigma = 0$  and  $\rho < 0$  all roots  $\tilde{\nu}_{1,2,3} \geq 0$  provided that  $\eta \geq \xi^2$ .
- b). if  $\sigma = 0$  and  $16/\xi^2 > \rho > 0$ , all  $\tilde{\nu}_{1,2,3} \geq 0$  if  $\xi^2 \geq \eta \geq (1 + \rho/16)\xi^2/(1 - \rho\xi^2/16)$ .
- c). if  $\sigma = 0$ , for  $\rho \geq 16/\xi^2$  there is no solution for  $\eta$  for which all  $\tilde{\nu}_{1,2,3} > 0$ .
- d). if  $\sigma = \pm 1$  all roots are positive if one respects simultaneously the following conditions:  $\eta \geq 16(\rho\xi^2/16 - 1)/15$  and  $-\eta^2 + \eta(15 - \rho\xi^2) - \xi^2(17 + \rho) \geq 0$  and  $-\eta^2 - \eta\rho\xi^2 + \rho\xi^4 \geq 0$ . In this last case, the solution for  $\eta$  exists and depends on the exact values of  $\rho$  and  $\xi$ .

For example, if  $\rho = 1$  and  $|\xi| \ll 1$ ,  $-0.38\xi^2 \leq \eta \leq 0.62\xi^2$ . Thus, if both effective  $\square$  operators are present, a UV theory, free of ghosts exists (all roots are positive). For  $\rho = 0$ , i.e. no  $\Phi_1^\dagger \square \Phi_1$  in original  $\mathcal{L}$ , there always exists a negative root. As a result, if an  $\mathcal{L}$  contains only the effective operator  $\int d^2\theta \Phi_1 \square \Phi_1$  there is *no ghost-free, UV theory*.

From the above cases we select those when all roots are positive. Introduce  $\Phi'_i = \tilde{u}_{ij}\Phi_j$  where  $\text{diag}\{\tilde{\nu}_1, \tilde{\nu}_2, \tilde{\nu}_3\} = \tilde{u} \tilde{k} \tilde{u}^\dagger$  and  $\tilde{k}_{ij}$  is the matrix of coefficients of the D-term (kinetic terms) in eq.(20). After rescaling  $\Phi'_i = \tilde{\Phi}_i/\sqrt{\tilde{\nu}_i}$  and with  $\tilde{z}_{kj} = (1/4)m\xi\tilde{u}_{j2}^*\tilde{u}_{k3}^*$ , one has

$$\mathcal{L} = \int d^4\theta \left[ \tilde{\Phi}_1^\dagger \tilde{\Phi}_1 + \tilde{\Phi}_2^\dagger \tilde{\Phi}_2 + \tilde{\Phi}_3^\dagger \tilde{\Phi}_3 \right] + \left\{ \int d^2\theta \left[ W \left( \frac{\tilde{u}_{j1}^* \tilde{\Phi}_j}{\sqrt{\tilde{\nu}_j}} \right) + \frac{\tilde{z}_{kj} \tilde{\Phi}_j \tilde{\Phi}_k}{\sqrt{\tilde{\nu}_j \tilde{\nu}_k}} \right] + \text{h.c.} \right\} \quad (25)$$

The coefficients  $\tilde{u}_{ij}$  and  $\tilde{z}_{jk}$  are calculable and depend on  $\rho$ ,  $\sigma$  and  $\xi$ .  $\mathcal{L}$  provides one possible UV theory<sup>10</sup> of the original Lagrangian eqs.(5), (8), is free of ghosts (under the above assumptions for  $\rho, \sigma, \xi$ ), and recovers the initial effective Lagrangian.

This method of identifying the UV theory can be extended to more complicated  $K$  and  $W$ . For this, we notice that ghosts superfields emerge if *kinetic mixing* of the superfields or bilinear derivative F-terms are present. In their absence, even if higher derivative (interaction) terms exist, no ghost are generated, provided that the (scalar) fields present in the interactions are not developing vev's. If this is not true, the analysis is complicated since interaction terms can contribute to the two-point Green function and lead to ghost superfields. In such case one could expand about the new ground state to identify their contributions to the kinetic terms to see if any ghosts superfields are generated.

<sup>10</sup>This is also a UV completion, since the UV theory is also renormalizable.

### 2.3 “Unfolding” a higher-derivative ghost-free chiral theory

In this section we study a different theory with higher-derivatives that is known to be ghost-free, since it does not introduce additional degrees of freedom (poles in the propagator) [19, 20, 21, 22]. Its Lagrangian is

$$\begin{aligned}\mathcal{L} &= \int d^4\theta \left\{ \Phi^\dagger \Phi + \frac{\rho}{\Lambda^4} D^\alpha \Phi D_\alpha \Phi \bar{D}_{\dot{\alpha}} \Phi^\dagger \bar{D}^{\dot{\alpha}} \Phi^\dagger \right\} \\ &= \int d^4\theta \left\{ \Phi^\dagger \Phi + \frac{\rho}{\Lambda^4} \left| \Phi D^2 \Phi - \frac{1}{2} D^2 \Phi^2 \right|^2 \right\} + \mathcal{O}(1/\Lambda^5) .\end{aligned}\quad (26)$$

where the sign of  $\rho$  is not yet fixed. This effective action has interactions of the form  $|\partial\phi|^4$  and higher-order algebraic terms for the chiral auxiliary field of the form  $|F|^4$ , but no dynamics for  $F$  is generated. With the help of four chiral Lagrange (super)field multipliers  $\Sigma_{1,2,3,4}$  one can rewrite (26) as

$$\begin{aligned}\mathcal{L} &= \int d^4\theta \left\{ \Phi^\dagger \Phi + \frac{\rho}{\Lambda^4} \left| \Phi D^2 \Phi - \frac{1}{2} D^2 \Phi^2 \right|^2 \right\} \\ &+ \left\{ \int d^2\theta \left[ \Sigma_1 (m_1 \Sigma_2 - \epsilon \bar{D}^2 \Phi^\dagger) + \Sigma_3 (m_2 \Sigma_4 - (1/m_3) \bar{D}^2 \Phi^{\dagger 2}) \right] + \text{h.c.} \right\} + \mathcal{O}(1/\Lambda^5)\end{aligned}\quad (27)$$

Eliminating  $\Sigma_{1,2,3,4}$  recovers the previous Lagrangian while  $m_{1,2,3}$  and  $\epsilon$  are included for dimensional reasons. Further

$$\begin{aligned}\mathcal{L} &= \int d^4\theta \left\{ \Phi^\dagger \Phi + 4\epsilon (\Sigma_1 \Phi^\dagger + \Sigma_1^\dagger \Phi) + \frac{\rho}{\Lambda^4} \left| \frac{m_1}{\epsilon} \Phi \Sigma_2^\dagger - \frac{m_2 m_3}{2} \Sigma_4^\dagger \right|^2 + \frac{4}{m_3} (\Sigma_3 \Phi^{\dagger 2} + \Sigma_3^\dagger \Phi^2) \right\} \\ &+ \left\{ \int d^2\theta (m_1 \Sigma_1 \Sigma_2 + m_2 \Sigma_3 \Sigma_4) + \text{h.c.} \right\} + \mathcal{O}(1/\Lambda^5).\end{aligned}\quad (28)$$

In this “unfolded” formulation, the parameters  $m_{1,2,3}$  and  $\epsilon$  are not physical, but they become so in the UV theory (see later). This form of  $\mathcal{L}$  has no higher derivative terms anymore. Given the high dimension of the initial derivative operator, the terms in eq.(28) that correspond to the initial operator, although polynomial, have mass dimension larger than four.

Despite its appearance, the Lagrangian in eq.(28) has no ghosts since it is equivalent to the original (ghost-free) Lagrangian in eq.(26). Indeed, for vanishing  $\text{vev}^{11} \langle \Phi \rangle = 0$ , it is obvious  $\Sigma_{2,3}$  have no dynamics and therefore only enforce constraints on  $\mathcal{L}$ . One of the constraints is that  $\Sigma_1$  is a composite field, so the apparent “off-diagonal” ghost-like kinetic term of  $\Sigma_1$  is actually a higher-order polynomial operator.

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<sup>11</sup>If this is not the case, the analysis is more complicated.

A UV theory of eq.(28) is found similar to the previous examples by adding the missing kinetic terms

$$\begin{aligned}\delta\mathcal{L}_{\text{kin.}} &= \int d^4\theta \left( \Sigma_1^\dagger \Sigma_1 + \Sigma_2^\dagger \Sigma_2 + \Sigma_3^\dagger \Sigma_3 \right), \\ \mathcal{L}_{\text{UV}} &= \mathcal{L} + \delta\mathcal{L}_{\text{kin.}}.\end{aligned}\tag{29}$$

The resulting UV Lagrangian is manifestly ghost-free for  $\epsilon < \frac{1}{4}$  and  $\rho > 0$ , which is the appropriate sign coming from general considerations [14]. Notice that in the UV Lagrangian of eq.(29) all parameters are physical, so the theory has now several mass scales. By integrating out the massive fields  $\Sigma_i$  one finds the corresponding effective action.

One can ask if this UV theory also generates additional effective operators beyond the original one in (26) of dimensions lower or equal to its dimension. The field equations of the massive fields determine their solution:

$$\begin{aligned}\Sigma_1 &= \frac{4\epsilon}{m_1^2} \left( 1 + \frac{\square}{m_1^2} \right) \square\Phi - \frac{\rho}{4\epsilon\Lambda^4} \bar{D}^2 (\Phi^\dagger D^\alpha \Phi D_\alpha \Phi) + \dots, \\ \Sigma_2 &= \frac{\epsilon}{m_1} \left( 1 + \frac{\square}{m_1^2} \right) \bar{D}^2 \Phi^\dagger + \dots, \\ \Sigma_3 &= \frac{2\rho m_3}{\Lambda^4} (\partial\Phi)^2 + \dots, \\ \Sigma_4 &= \frac{1}{m_2 m_3} \bar{D}^2 \Phi^{\dagger 2} + \frac{\rho m_3}{2 m_2 \Lambda^4} \bar{D}^2 (\partial\Phi^\dagger)^2 + \dots,\end{aligned}\tag{30}$$

where the dots stand for terms that contribute to operators of dimension higher than eight. Substituting this solution back in  $\mathcal{L}_{\text{UV}}$  of eq.(29) one finds a low energy, effective Lagrangian below

$$\mathcal{L}_{\text{eff}} = \int d^4\theta \left\{ \Phi^\dagger \Phi + \frac{16\epsilon^2}{m_1^2} \Phi^\dagger \square\Phi + \frac{16\epsilon^2}{m_1^4} \square\Phi^\dagger \square\Phi + \frac{\rho}{\Lambda^4} D^\alpha \Phi D_\alpha \Phi \bar{D}_{\dot{\alpha}} \Phi^\dagger \bar{D}^{\dot{\alpha}} \Phi + \dots \right\}.\tag{31}$$

We thus recovered our initial operator, the last term above, suppressed by  $\Lambda$ . The other two operators are suppressed by another mass scale  $\frac{m_1}{\epsilon}$ . One can in principle arrange that  $\Lambda \ll m_1/\epsilon$  by a suitable choice for (dimensionless)  $\epsilon$ . In this case our operator of interest in eq.(26) is the leading one generated at low energy.

As seen above, consistency of the UV theory we found demands that  $\rho = 1$ .

### 3 Effective operators for a linear multiplet

Another important supersymmetric multiplet, notably in string models, is a real linear multiplet  $L$  [32], containing a real scalar field  $c$ , a fermion  $\psi$  and an antisymmetric tensor  $b_{\mu\nu}$ . The multiplet satisfies the constraints

$$D^2 L = \overline{D}^2 L = 0 , \quad (32)$$

with a solution

$$L = D^\alpha \mathcal{Z}_\alpha + \overline{D}_{\dot{\alpha}} \overline{\mathcal{Z}}^{\dot{\alpha}} , \quad (33)$$

where  $\mathcal{Z}_\alpha$  is a chiral spinor superfield. The superspace expansion in the rigid case is

$$L = c + \theta\psi + \bar{\theta}\bar{\psi} - \theta\sigma^\mu\bar{\theta}\epsilon_{\mu\nu\rho\sigma}\partial^\nu b^{\rho\sigma} + \frac{i}{2}\theta^2\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi - \frac{i}{2}\bar{\theta}^2\theta\sigma^\mu\partial_\mu\bar{\psi} + \frac{1}{4}\theta^2\bar{\theta}^2\Box c . \quad (34)$$

The massless action for a linear multiplet has the gauge symmetry

$$\mathcal{Z}_\alpha \rightarrow \mathcal{Z}_\alpha - iW_\alpha , \quad (35)$$

if  $W_\alpha$  is a gauge superfield strength satisfying  $D^\alpha W_\alpha = \overline{D}_{\dot{\alpha}} \overline{W}^{\dot{\alpha}}$ . The free action of a massive linear multiplet is [33]

$$\mathcal{L} = \int d^4\theta \left( -\frac{1}{2}L^2 \right) + \left\{ \int d^2\theta \frac{-\Lambda^2}{2} \mathcal{Z}^\alpha \mathcal{Z}_\alpha + \text{h.c.} \right\} . \quad (36)$$

#### 3.1 “Unfolding” effective operators in the linear multiplet sector

In applications one can encounter an effective operator of the type shown below acting in the linear multiplet sector

$$\mathcal{L}_0 = \int d^4\theta \left[ -\frac{1}{2}L^2 + \frac{\rho}{\Lambda^2} L \Box L \right] + \mathcal{O}(1/\Lambda^3) \quad (37)$$

where  $\rho = \pm 1$  was introduced to allow either sign for the last term.  $\mathcal{L}_0$  can be written as

$$\mathcal{L}_0 = \int d^4\theta \left[ -\frac{1}{2}L^2 + \frac{\rho}{8\Lambda^2} L D^\alpha \bar{D}^2 D_\alpha L \right] + \mathcal{O}(1/\Lambda^3) , \quad (38)$$

by using the identity  $\overline{D}_{\dot{\alpha}} D^2 \bar{D}^{\dot{\alpha}} = D^\alpha \bar{D}^2 D_\alpha = \frac{1}{2}\{D^2, \bar{D}^2\} + 8\Box$ . The above effective operator

can be “unfolded” by using our experience so far<sup>12</sup>. The result can be guessed directly, by starting with

$$\mathcal{L} = \int d^4\theta \left[ -\frac{1}{2}L^2 + aLL' \right] + \left\{ \int d^2\theta \frac{-\Lambda^2}{2} \mathcal{Z}'^\alpha \mathcal{Z}'_\alpha + \text{h.c.} \right\}, \quad (39)$$

with a constraint  $L' = D^\alpha \mathcal{Z}'_\alpha + \overline{D}_{\dot{\alpha}} \overline{\mathcal{Z}}'^{\dot{\alpha}}$ ; here  $L'$  is a massive linear multiplet.  $a$  is a real dimensionless numerical coefficient to be identified shortly. The above Lagrangian has one standard linear multiplet ( $L$ ) and an additional, ghost linear multiplet since the determinant of the kinetic terms matrix is negative  $\propto -a^2 < 0$ . One can eliminate the massive linear multiplet  $L'$  via its equation of motion that is actually obtained from that for the unconstrained (independent) field  $\mathcal{Z}'$

$$\mathcal{Z}'_\alpha = \frac{a}{4\Lambda^2} \overline{D}^2 D_\alpha L \quad \Rightarrow \quad L' = \frac{a}{4\Lambda^2} (D^\alpha \overline{D}^2 D_\alpha + \overline{D}_{\dot{\alpha}} D^2 \overline{D}^{\dot{\alpha}}) L. \quad (40)$$

The effective Lagrangian after eliminating  $L'$  becomes

$$\mathcal{L} = \int d^4\theta \left[ -\frac{1}{2}L^2 + \frac{2a^2}{\Lambda^2} L \square L \right], \quad (41)$$

This is identical to eq.(37) provided that  $a^2 = \rho/2$  which has a solution for  $\rho = +1$  only. With this value,  $\mathcal{L}$  of eq.(39) is an “unfolded” version of initial  $\mathcal{L}_0$  of eq.(37) since it has no higher dimensional operators, but it contains an additional ghost-like superfield.

### 3.2 Ghost-free UV theory for the linear multiplet case

A natural UV theory of the effective Lagrangian above is to add to eq.(39) a kinetic term for the massive linear multiplet  $L'$

$$\mathcal{L} = \int d^4\theta \left[ -\frac{1}{2}L^2 - \frac{1}{2}L'^2 + aLL' \right] + \left\{ \int d^2\theta \frac{-\Lambda^2}{2} \mathcal{Z}'^\alpha \mathcal{Z}'_\alpha + \text{h.c.} \right\}, \quad (42)$$

$\mathcal{L}$  is ghost-free (if  $a^2 < 1$ ) and recovers the Lagrangian  $\mathcal{L}_0$  plus higher derivative operators.

Other UV theories are possible. Consider for example the coupling of the massless linear multiplet  $L$  to a massive vector multiplet  $V$

$$\mathcal{L}_1 = \int d^4\theta \left( -\frac{1}{2}L^2 - M'LV + \frac{M^2}{2}V^2 \right) + \left\{ \int d^2\theta \frac{1}{4}W^\alpha W_\alpha + \text{h.c.} \right\}. \quad (43)$$

---

<sup>12</sup> According to our method the constraint  $L' = D^\alpha \mathcal{Z}'_\alpha + \overline{D}_{\dot{\alpha}} \overline{\mathcal{Z}}'^{\dot{\alpha}}$  should be imposed via a Lagrange multiplier superfield. We impose this constraint directly when using the eq of motion of  $\mathcal{Z}'$  instead of (constrained)  $L'$ .

The field equation of the massive vector multiplet leads to

$$V = \frac{M'}{M^2 + (1/4) D^\alpha \bar{D}^2 D_\alpha} L = \frac{M'}{M^2 + 2\Box} L, \quad (44)$$

which, when inserted back into eq.(43), leads to

$$\mathcal{L}_{1,\text{eff}} = \int d^4\theta \left[ -\frac{1}{2} L^2 - \frac{M'^2}{2} L \frac{1}{M^2 + 2\Box} L \right] \quad (45)$$

This Lagrangian is equivalent to the original effective Lagrangian  $\mathcal{L}_0$  of eq.(37) after an expansion in  $\Box/M'$  and a wave function renormalization of  $L \rightarrow L/(1 + M'^2/M^2)^{1/2}$  and with the identification  $\Lambda \equiv M(1 + M^2/M'^2)^{1/2}$ . Like in the previous example,  $\rho = +1$ .

## 4 Effective operators in the gauge sector

### 4.1 “Unfolding” the effective operators in the gauge sector

The above analysis can be extended to cases when such effective operators are present in the gauge sector. Without restriction to generality, we consider an Abelian case with<sup>13</sup>

$$\mathcal{L} = \int d^4\theta K(\Phi^i, e^V, \Phi_j^\dagger, \dots) + \left\{ \int d^2\theta \left[ \frac{1}{4} W^\alpha W_\alpha - \frac{\rho}{\Lambda^2} W^\alpha \Box W_\alpha \right] + \text{h.c.} \right\} + \mathcal{O}(1/\Lambda^3) \quad (46)$$

where  $\rho = \pm 1$ . With  $W_\alpha = -(1/4) \bar{D}^2 D_\alpha V$  then

$$\delta\mathcal{L} = -\frac{\rho}{\Lambda^2} \int d^2\theta W^\alpha \Box W_\alpha = -\frac{\rho}{4\Lambda^2} \int d^4\theta W^\alpha D^2 W_\alpha = \frac{-\rho}{2\Lambda^2} \int d^4\theta (D^\alpha W_\alpha)^2 \quad (47)$$

where we used that  $D^2 \epsilon^{\gamma\alpha} = -2D^\gamma D^\alpha$ . Then

$$\mathcal{L} = \int d^4\theta \left[ K(\Phi^i, e^V, \Phi_j^\dagger) - \frac{\rho}{2\Lambda^2} [(D^\alpha W_\alpha)^2 + (\bar{D}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}})^2] \right] + \left\{ \int d^2\theta \frac{1}{4} W^\alpha W_\alpha + \text{h.c.} \right\} + \mathcal{O}\left[\frac{1}{\Lambda^3}\right] \quad (48)$$

As before, we introduce an auxiliary (real) superfield  $V'$  which enables us to remove the higher dimensional (derivative) operator via a constraint

$$D^\alpha W_\alpha = m^2 V' \quad (49)$$

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<sup>13</sup> We denote the gauge field strength by  $W^\alpha$ , not to be confused with the superpotential  $W$ .

Here  $m$  is a small arbitrary scale of the theory which is set to 0 at the end of the calculation. We implement the constraint using a Lagrangian multiplier which is a real superfield  $\Sigma$ , as shown below:

$$\begin{aligned}\mathcal{L} &= \int d^4\theta \left[ K(\Phi^i, e^V, \Phi_j^\dagger) - \frac{\rho m^4}{\Lambda^2} V'^2 + 2\Sigma (D^\alpha W_\alpha - m^2 V') \right] \\ &+ \left\{ \int d^2\theta \frac{1}{4} W^\alpha W_\alpha + \text{h.c.} \right\} + \mathcal{O}(1/\Lambda^3)\end{aligned}\quad (50)$$

Using the Lagrangian in eq.(50), the eq of motion for  $\Sigma$  reproduces the constraint eq.(49).  $V'$  can be eliminated (integrated out exactly) since its equations of motion are algebraic, so  $\mathcal{L}$  becomes:

$$\begin{aligned}\mathcal{L} &= \int d^4\theta \left[ K(\Phi^i, e^V, \Phi_j^\dagger) + \rho \Lambda^2 \Sigma^2 \right] \\ &+ \left\{ \int d^2\theta \left[ \frac{1}{4} W^\alpha W_\alpha - W^\alpha(\Sigma) W_\alpha \right] + \text{h.c.} \right\} + \mathcal{O}(1/\Lambda^3)\end{aligned}\quad (51)$$

where  $W^\alpha(\Sigma) = -\frac{1}{4}\overline{D}^2 D^\alpha \Sigma$ . We obtained a second-order theory with renormalizable interactions (by power counting) with the original vector superfield  $V$  and an additional massive one  $\Sigma$ . The new field  $\Sigma$  is a ghost (vector) superfield since the determinant of the kinetic terms is negative. The gauge kinetic mixing can be diagonalised by a rotation and an appropriate rescaling. Finally, one can also integrate out  $\Sigma$  using the above  $\mathcal{L}$  to obtain an effective operator of second order, as done for the matter sector, eqs.(17), (18). The result depends on the structure assumed for  $K$  (for example one can consider the simplest case  $\Phi^\dagger e^V \Phi$ , etc).

## 4.2 A ghost-free UV completion in the gauge sector

Can we find in this case a simple, ghost-free (renormalizable) UV theory of  $\mathcal{L}$  in eq.(46), (51)? From the matter sector we know that simply adding a positive kinetic term, in this case for  $\Sigma$  in eq.(51), is the way to proceed. We thus add

$$\delta\mathcal{L} = \int d^2\theta \delta W^\alpha(\Sigma) W_\alpha(\Sigma) + \text{h.c.}\quad (52)$$

$\delta\mathcal{L}$  together with  $\mathcal{L}$  of eq.(51) can be brought to a diagonal basis after a suitable rotation applied to  $V, \Sigma$ . Then one chooses values for  $\delta$  so that there are no ghost vector superfields in  $\mathcal{L} + \delta\mathcal{L}$ . This new Lagrangian gives a UV theory that is also renormalizable (UV completion).

### 4.3 Application: an example generating Dirac gaugino masses

As discussed, not all theories with  $\mathcal{L}$  having powers of superderivatives generate ghosts, if these are present in interactions. We construct another example in the following. Start with a model

$$\mathcal{L} = \int d^4\theta \left[ \frac{\Lambda'^2}{2} V'^2 + X^\dagger e^{V'} X + \Phi^\dagger \Phi \right] + \left\{ \int d^2\theta \left[ \frac{1}{4} W^\alpha W_\alpha + \frac{1}{\Lambda} W'^\alpha W_\alpha \Phi + W(\Phi) \right] + \text{h.c.} \right\} \quad (53)$$

where  $\Lambda'$  is large, comparable to  $\Lambda$ . We have a massive gauge field  $V'$  of field strength  $W'_\alpha$ , a field  $X$  charged under it, a gauge kinetic term for  $V$  and an interaction term with  $\Phi$  neutral under  $V, V'$ . If  $\Phi$  has a scalar component with non-zero vev, then we would have kinetic mixing  $W W'$  and this would induce the presence of a ghost (for  $W'$ ). However, assume that the scalar component of  $\Phi$  has a vanishing vev, ensured by a suitable choice of the superpotential  $W(\Phi)$ . Then the field equation for  $V'$  gives  $V'(\Lambda'^2 + X^\dagger X) = 1/\Lambda [D^\alpha(W_\alpha \Phi) + \text{h.c.}] - X^\dagger X$ . Using this in  $\mathcal{L}$ , we obtain an effective Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{eff}} \supset & \int d^4\theta \left[ \Phi^\dagger \Phi + X^\dagger X - \frac{1}{2\Lambda'^2} \left( 1/\Lambda [D^\alpha(W_\alpha \Phi) + \text{h.c.}] - X^\dagger X \right)^2 \right] \\ & + \int d^2\theta \left[ \frac{1}{4} W^\alpha W_\alpha + W(\Phi) \right] + \text{h.c.} \end{aligned} \quad (54)$$

up to additional terms (not shown) suppressed by extra powers  $\Lambda'$ . If we identify the chiral superfield  $X$  with the spurion of supersymmetry breaking, then  $\mathcal{L}_{\text{eff}}$  contains a Dirac mass term [34, 35]

$$\mathcal{L}_{\text{eff}} \supset \frac{1}{\Lambda'^2 \Lambda} \int d^4\theta \left[ X^\dagger X D^\alpha(W_\alpha \Phi) + \text{h.c.} \right] \quad (55)$$

This is seen by considering the fermionic component of  $\Phi$  and the gaugino  $\lambda$  in  $W_\alpha$  giving a mass term for  $\lambda \psi$  where  $\psi$  is the Weyl fermion of  $\Phi$ . Finally, a UV theory of Lagrangian (53) is obtained by adding there an F-term

$$\delta \mathcal{L} = \frac{1}{4} \int d^2\theta W'^\alpha W'_\alpha + \text{h.c.} \quad (56)$$

We obtain in this way a two-derivative, ghost free Lagrangian that generates in the low energy the Dirac gaugino mass term.

#### 4.4 Supersymmetric Euler-Heisenberg Lagrangian

Another interesting case is that of a supersymmetric generalisation of the Euler-Heisenberg Lagrangian which is a higher-derivative gauge theory that can be ghost-free<sup>14</sup>. It is given by

$$\mathcal{L} = \int d^4\theta \frac{\rho}{\Lambda^4} W^\alpha W_\alpha \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} + \left\{ \int d^2\theta \frac{1}{4} W^\alpha W_\alpha + \text{h.c.} \right\} \quad (57)$$

where  $\rho = \pm 1$  accounts for possible signs of this operator.  $\mathcal{L}$  contains the gauge field term

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\rho}{4\Lambda^4} \left[ (F_{\mu\nu} F^{\mu\nu})^2 + (F_{\mu\nu} \tilde{F}^{\mu\nu})^2 \right] + \dots, \quad (58)$$

One must have  $\rho > 0$  according to the constraints discussed in [14]. Similar to the chiral superfields example discussed earlier,  $\mathcal{L}$  can be re-formulated as (“unfolded” into) a second-order theory. The idea is to introduce a constraint that replaces a pair  $W^\alpha W_\alpha$  of the D-term by a chiral superfield so this term becomes quadratic (in  $S$ ) and thus “removes” the extra derivatives. We thus introduce two additional chiral superfields  $\Phi$  and  $S$ , according to

$$\mathcal{L} = \int d^4\theta S^\dagger S + \left\{ \int d^2\theta \left[ \frac{1}{4} W^\alpha W_\alpha + M \Phi (S - \epsilon W^\alpha W_\alpha) \right] + \text{h.c.} \right\}, \quad (59)$$

where  $\Phi$  is the Lagrange multiplier (chiral superfield) that implements the constraint and  $M$  is an arbitrary mass scale. Notice that, up to total derivatives, there is a continuous shift symmetry  $\Phi \rightarrow \Phi + i\alpha$ , where  $\alpha$  is a real parameter. After integrating out  $S$  and  $\Phi$  one recovers  $\mathcal{L}$  of (57) provided that  $\rho/\Lambda^4 = \epsilon^2$ . It is interesting that this second-order formulation of the starting  $\mathcal{L}$  is actually consistent with the condition  $\rho > 0$  [14].

The situation is however more subtle because the chiral operator  $W^\alpha W_\alpha$  is constrained by  $W^\alpha W_\alpha = \frac{1}{2} \bar{D}^2 \Omega$ , where the real superfield  $\Omega$  is the Chern-Simons superfield<sup>15</sup> [37, 38]. Accordingly, the field  $S$  in eq.(59) is not an independent chiral superfield, but the field strength of a three-form superfield  $S = -\frac{1}{4} \bar{D}^2 U$ , where  $U$  is a dimensionless superfield<sup>16</sup>.

Let us briefly review some details regarding the real three-form multiplet  $U$ . This is defined in superspace by

$$U = \bar{U} = B + i(\theta\chi - \bar{\theta}\bar{\chi}) + \theta^2 \bar{s} + \bar{\theta}^2 s + \frac{1}{3} \theta \sigma^m \bar{\theta} \epsilon_{mnpq} C^{npq} + \theta^2 \bar{\theta} (\sqrt{2} \bar{\lambda} + \frac{1}{2} \bar{\sigma}^m \partial_m \chi) + \bar{\theta}^2 \theta (\sqrt{2} \lambda - \frac{1}{2} \sigma^m \partial_m \bar{\chi}) + \theta^2 \bar{\theta}^2 (D + \frac{1}{4} \square B). \quad (60)$$

<sup>14</sup> Here we “unfold” this operator at the classical level. However this operator can also be generated by loop corrections in a standard gauge theory. For studies of this operator in supergravity, see e.g. [36].

<sup>15</sup> In the Abelian case, it is given explicitly by  $\Omega = -\frac{1}{2} (D^\alpha W_\alpha + \bar{D}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} + V D^\alpha W_\alpha)$ .

<sup>16</sup> This is similar to the dilaton case, for example.

The difference between a three-form multiplet  $U$  and a regular vector superfield  $V$  is the replacement of the vector potential  $V^m$  by the three-form  $C^{mpq}$ . To find the appropriate kinetic terms, the analog of the chiral field strength superfield  $W_\alpha$  for a vector multiplet is the chiral superfield [39, 37]

$$S = -\frac{1}{4}\overline{D}^2 U \quad , \quad S(y, \theta) = s + \sqrt{2}\theta\lambda + \theta^2(D + iF) \quad , \quad (61)$$

with  $F$  defined by  $F = \frac{1}{4!}\epsilon_{mnpq}F^{mnpq}$ , where  $F^{mnpq}$  is the four-form field strength. While the massless three-form multiplet  $U$  has only two propagating bosonic ( $s$ ) and two fermionic ( $\lambda$ ) degrees of freedom, a massive three-form multiplet has four bosonic and fermionic degrees of freedom. Notice also that a massless three-form multiplet has the gauge invariance  $U \rightarrow U - L$ , where  $L$  is a linear multiplet. This symmetry is broken by a mass term, similarly to the case of a standard vector multiplet  $V$ . References [39, 37] contain detailed explanations about the three-form multiplet.

Returning to eq.(59), there is a dual formulation of this equation, which can be found by starting from the master Lagrangian

$$\mathcal{L} = \int d^4\theta \left[ S^\dagger S + M^2 Q(U - L + 2\epsilon\Omega) \right] + \left\{ \int d^2\theta \frac{1}{4} W^\alpha W_\alpha + \text{h.c.} \right\} \quad , \quad (62)$$

where  $Q$  is a real superfield and  $L$  a (dimensionless) linear multiplet. The equation of motion for  $L$  gives  $MQ = \Phi + \Phi^\dagger$ , which when used into eq.(62) recovers the action of eq.(59). Alternatively, eliminating  $Q$  one obtains

$$U = L - 2\epsilon\Omega \quad , \quad S = \epsilon W^\alpha W_\alpha \quad , \quad (63)$$

which inserted into (62) does recover the starting Lagrangian of eq.(57) (with  $\epsilon^2 = \rho/\Lambda^4 > 0$ ).

Finding the UV theory of the Euler-Heisenberg Lagrangian is now done by adding the missing kinetic term in the chiral formulation

$$\mathcal{L}_{\text{UV}} = \int d^4\theta \left[ S^\dagger S + \frac{1}{2}(\Phi + \Phi^\dagger)^2 \right] + \left\{ \int d^2\theta \left[ \frac{1}{4} W^\alpha W_\alpha + M\Phi(S - \epsilon W^\alpha W_\alpha) \right] + \text{h.c.} \right\} \quad . \quad (64)$$

The shift symmetric kinetic term for  $\Phi$  is equivalent to a standard canonical one in rigid SUSY, but is different in the supergravity version. The dual UV Lagrangian is found by introducing a vector multiplet as above and eliminating it out via its equation of motion. The result is

$$\mathcal{L} = \int d^4\theta \left[ S^\dagger S - \frac{M^2}{2} (U - L + 2\epsilon\Omega)^2 \right] + \left\{ \int d^2\theta \frac{1}{4} W^\alpha W_\alpha + \text{h.c.} \right\} , \quad (65)$$

and contains a massive three-form multiplet. The action in eq.(65) is fully gauge invariant. In particular, under  $U(1)$  gauge transformations and in the unitary gauge  $L = 0$ , the relevant transformations are

$$\delta V = \Theta + \bar{\Theta} \quad , \quad \delta U = \epsilon \left[ D^\alpha (\Theta W_\alpha) + \bar{D}_{\dot{\alpha}} (\Theta^\dagger \bar{W}^{\dot{\alpha}}) \right] . \quad (66)$$

This discussion neglected subtleties related to the existence of boundary terms in the action. While they are important to obtain a fully consistent Lagrangian (see e.g. [40]), they are not relevant for the above discussion.

It is interesting to note that the Lagrangian in eq.(65) (with the gauge field set to zero) is that of the chaotic inflationary model described in the last reference in [40].

## 5 More general cases

The method of superfield constraints that we introduced can be generalised to more arbitrary  $K, W$  which have as arguments *chiral* functions

$$\mathcal{L} = \int d^4\theta \ K(\Phi_j, \Phi_j^\dagger, \bar{D}^2 \Phi_j^\dagger, D^2 \Phi_j, D^2 \bar{D}^2 \Phi_k^\dagger \dots) + \left\{ \int d^2\theta \ W(\Phi_j, \bar{D}^2 \Phi_j^\dagger, \bar{D}^2 D^2 \Phi_k \dots) + \text{h.c.} \right\} \quad (67)$$

Note the dependence on the superderivatives of many fields  $\Phi_j$ . The dots stand for powers of such superderivatives that may also be present. All higher dimensional terms are suppressed by appropriate powers of a high scale (the Kahler curvature tensor). This action is difficult to compute and investigate in component fields. However, one can introduce constraints  $\bar{D}^2 \Phi_i^\dagger = m \tilde{\Phi}_i$ ,  $i = 1, 2, \dots$ , and similar for the other, higher order superderivatives that may be present, by using an iterative procedure (as done in the Appendix). These constraints are then added to the original  $\mathcal{L}$  as F-terms of type

$$\frac{m}{\Lambda} \int d^2\theta \sum_{j \geq 1} (\bar{D}^2 \Phi_j^\dagger - m \tilde{\Phi}_j) \Sigma_j + \text{h.c.} \quad (68)$$

with  $\Sigma_j$  the Lagrange multipliers chiral superfields whose eqs of motion recover the constraints. As in previous cases, the coefficient in front of the integral is useful to ensure the constraint is vanishing in the limit  $\Lambda \rightarrow \infty$  and that the kinetic mixing is under control. Similar considerations apply for the vector (gauge) or linear multiplet sectors. In this way one obtains a  $K$  and  $W$  that only depend on the superfields  $\chi_k = \{\Phi_i, \tilde{\Phi}_j, \Sigma_l\}$  but not on the superderivatives. With  $K = K(\chi_k, \chi_k^\dagger)$  and  $W(\chi_k)$ , the Grassmann integrals in  $\mathcal{L}$  can

then be performed as for the non-linear sigma-model by using a (Grassmann space) Taylor expansion, to find the component fields action [41]. This will be a second order theory, but with additional fields (that can then be eliminated by the field equations, if massive enough).

The “unfolding” method of superfield constraints applied to higher order terms in momentum expansion of the Lagrangian can also be applied to other general cases. One may be interested in cases when the momentum is closer to the effective cutoff, when even higher order terms (beyond those considered here) are relevant. In the Appendix we present the extreme case of including *all orders* in such an expansion and apply our method. To illustrate it, we simplify the analysis and use instead a *known UV-complete* (renormalizable) theory, eq.(A-1) and integrate a massive state of mass  $\Lambda$  to all orders in  $\partial/\Lambda$ , to generate such a momentum expansion. The new theory, eq.(A-8) is non-local [2] equivalent to the initial one, eq.(A-1). We then use an infinite set of superfield constraints, eqs.(A-10) to eliminate all powers of the superderivatives. This shows that even in this case there still exists an “unfolding” formulation of the higher order theory, in which the infinite series (in powers of  $\partial/\Lambda$ ) of effective operators is replaced by polynomial (quadratic) terms in superfields, plus extra superfields that are massive (mass of order  $\Lambda$ ), see eqs.(A-13), (A-18). Half of these massive superfields are ghost-like and half of them are particle-like. Truncating this theory to a given number of superfields is equivalent to truncating the initial Lagrangian to a given power of  $\partial/\Lambda$ . In this “truncated” theory, one can integrate out these massive fields to obtain a Lagrangian with new effective operators polynomial in superfields, as in eq.(17), (18). While this example is very simple, it shows that our method can be extended to higher or all orders in momentum expansion by using an appropriate set of superfield constraints and their iteration.

## 6 Conclusions

Effective field theories contain a series in momentum expansion that includes a special class of higher dimensional operators. These are generated by classical integration of massive states even in renormalizable, UV complete theories (and also by quantum corrections of compactification). The result of such integration is in general truncated to a given order in momentum expansion  $\partial/\Lambda$  ( $\Lambda$  is the effective cutoff). This leads to an effective theory with higher derivative operators that can induce the presence of (super)fields of negative kinetic terms (ghosts). Contrary to a common perception, there is nothing “pathological” about their presence here. They are just artefacts of the expansion in  $\partial/\Lambda$  obtained after integrating out massive states (of mass  $\sim \Lambda$ ) of the UV theory. Their presence and that of the corresponding (derivative) operators is not problematic and we showed how to treat them. Finally, keeping all orders in the momentum expansion gives a non-local theory equivalent (classically) to the

initial, fundamental theory that generated the effective operators in the low energy.

In a manifestly supersymmetric approach and using superfield constraints, we showed how such effective operators acting on chiral, vector and linear superfields can be “unfolded” into “standard” operators, polynomial in superfields, in the order of truncation in  $\partial/\Lambda$  considered. To see this, consider the case of chiral superfields. In such case one replaced the superderivatives  $\overline{D}^2\Phi_0^\dagger$  by new chiral superfields  $m\chi = \overline{D}^2\Phi_0^\dagger$  where  $m$  is a small, arbitrary mass introduced for dimensional reasons. This chiral superfield constraint was enforced with a Lagrange multiplier chiral superfield and in this way all superderivatives are eliminated. The method can be applied and iterated to higher orders in  $D, \overline{D}$  (and also to non-derivative operators). After this “unfolding” there are no superderivatives left but only terms polynomial in superfields and additional superfields (some of which are ghost-like) of a mass of order  $\Lambda$ . These can be integrated out to obtain a ghost-free low energy effective action that is polynomial in superfields and corresponding to the order in  $\partial/\Lambda$  considered. The action so obtained can then be used for phenomenology. This procedure can be repeated to higher orders in  $\partial/\Lambda$  for improved accuracy. The method was then applied to cases when superderivative operators act in the gauge and linear multiplets sectors, with similar results.

The “unfolding” method of superfields constraints helps one identify (two-derivative and ghost-free) UV theories that generate at low-energy the effective operators considered. We applied this method to the case when  $\square$ -operators acted on chiral, vector or linear superfields, and using their “unfolded” formulation we identified the associated UV theory (not necessarily unique). In these examples the initial effective operators were *quadratic* in fields or gauge fields strengths.

Further examples were provided of ghost-free effective theories with higher dimensional (derivative) *interactions*: a chiral superfield model with such an interaction operator, a model generating at low-energy Dirac-gaugino masses and an effective model that is a supersymmetric version of the Euler-Heisenberg Lagrangian. Each model was “unfolded” into a second-order theory for which we subsequently identified a UV formulation (*i.e.* with two-derivatives only and ghost-free).

We also showed how our method can be extended to any  $K$  and  $W$  as arbitrary functions of chiral (functions) arguments. Finally, the Appendix provided a special case showing how our method can be used to all orders in momentum expansion. This is relevant for momenta closer to the effective cutoff. This was possible by using an iteration procedure and an infinite set of superfield constraints, with similar conclusions as in the examples quadratic in fields.

## A Appendix: “Unfolding” effective operators to all orders

In this Appendix we show that the “unfolding” method of using superfield constraints to eliminate higher powers of the superderivatives can be extended to all orders in the momentum expansion of an effective theory.

The plan of this section is as follows. In Section A.1, we choose a simple model for which the UV completion is known, integrate a massive state  $\chi$  *exactly* (to all orders), to generate an infinite series in momentum expansion and a non-local theory, see eqs.(A-1), (A-6), (A-8). In section A.2 this theory truncated to an arbitrary order ( $n$ ) is “unfolded” into a traditional second-order theory that is shown to have only polynomial terms in superfields and an additional set of massive superfields (half of which are ghost-like), eqs.(A-13), (A-18). This theory is classically equivalent to the starting one of eqs.(A-6), (A-8) in the corresponding order in  $\partial/\Lambda$ . When integrating these massive superfields one can then generate an effective Lagrangian of second-order, with only polynomial effective operators as done in the text, eqs.(17), (18).

### A.1 Effective operators from a renormalizable theory: all orders analysis

Consider a simple renormalizable, UV complete Lagrangian of eq.(1)

$$\mathcal{L} = \int d^4\theta \left\{ \Phi_0^\dagger \Phi_0 + \chi^\dagger \chi \right\} + \left\{ \int d^2\theta \left[ (1/8) \Lambda \chi^2 + (1/4) m \Phi_0 \chi + W(\Phi_0) \right] + h.c. \right\} \quad (\text{A-1})$$

so  $\Lambda$  is here the mass of  $\chi$ . Ignoring contributions from  $W$  (if any), the masses of the scalar components of  $\chi$ ,  $\Phi_0$  are  $(1/8) \Lambda (1 \mp [1 + 4m^2/\Lambda^2]^{1/2})$ . For  $\Lambda \gg m$  we can integrate out  $\chi$  via its eq of motion

$$-\overline{D}^2 \chi^\dagger + \Lambda \chi + m \Phi_0 = 0 \quad (\text{A-2})$$

which has a (iterative) solution

$$\chi = \frac{-m}{\Lambda} \Phi_0 + \frac{-m}{\Lambda^2} \overline{D}^2 \Phi_0^\dagger + \frac{-m}{\Lambda^3} \overline{D}^2 D^2 \Phi_0 + \frac{-m}{\Lambda^4} \overline{D}^2 D^2 \overline{D}^2 \Phi_0^\dagger + \dots \quad (\text{A-3})$$

This solution is used back in  $\mathcal{L}$  to integrate  $\chi$ . Due to technical difficulties, one always truncates such expansion to a fixed (low) order. One finds for example

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \int d^4\theta \Phi_0^\dagger [1 - \xi^2 \square_*] \Phi_0 \\ & + \int d^2\theta \left[ \frac{-m\xi}{8} ((1 + \xi^2)^{-1/2} \Phi_0)^2 + \frac{m\xi}{8} \Phi_0 \square_* \Phi_0 + W[\Phi_0/(1 + \xi^2)^{1/2}] \right] + h.c. + \mathcal{O}\left(\frac{1}{\Lambda^5}\right) \end{aligned} \quad (\text{A-4})$$

after a rescaling  $\Phi_0 \rightarrow \Phi_0/(1 + \xi^2)^{1/2}$  was made and where

$$\xi = \frac{m}{\Lambda}, \quad \square_* = \frac{\square}{(\Lambda/4)^2}, \quad \square\Phi_0 = -16\overline{D}^2 D^2 \Phi_0. \quad (\text{A-5})$$

Note the presence of the  $\square$  operators in the F- and D-terms investigated in the text, also leading to the presence of the extra ghost superfields.

But we can go beyond the approximation of truncated series. One finds the exact result

$$\begin{aligned} \mathcal{L} = & \int d^4\theta \Phi_0^\dagger \left[ 1 + \frac{m^2}{\Lambda^2} \left( 1 + \sum_{n \geq 1} \frac{1}{\Lambda^{2n}} [\overline{D}^2 D^2]^n \right) \right] \Phi_0 \\ & + \int d^2\theta \left[ \frac{-m^2}{8\Lambda} \Phi_0 \left( 1 + \sum_{n \geq 1} \frac{1}{\Lambda^{2n}} [\overline{D}^2 D^2]^n \right) \Phi_0 + W(\Phi_0) \right] + \text{h.c.} \end{aligned} \quad (\text{A-6})$$

or

$$\mathcal{L} = \int d^4\theta \Phi_0^\dagger \left[ 1 + \xi^2 \frac{1 - (-\square_*)^n}{1 + \square_*} \right] \Phi_0 + \left\{ \int d^2\theta \left[ \frac{-\Lambda}{8} \xi^2 \Phi_0 \frac{1 - (-\square_*)^n}{1 + \square_*} \Phi_0 + W(\Phi_0) \right] + \text{h.c.} \right\} \quad (\text{A-7})$$

where  $n \rightarrow \infty$ ; in this limit there is no ghost, but truncating  $\mathcal{L}$  to  $n$  finite generates ghosts (artefacts). One ignores the terms  $\square_*^n$  since integrating out  $\chi$  of mass of order  $\Lambda$  we effectively integrate momenta that are below this mass scale. Further, the F-term can be replaced, up to a total space-time integral as follows:  $\Phi_0 (1 + \square_*)^{-1} \Phi_0 \rightarrow [(1 + \square_*)^{-1/2} \Phi_0]^2$ . To see this expand  $(\square_* + 1)^{-1}$  and use repeated integration by parts. One then rescales  $\Phi \rightarrow Z^{1/2} \Phi$ ,  $\Phi^\dagger \rightarrow \Phi^\dagger Z^{1/2}$  where  $Z^{-1} = 1 + \xi^2 (\square_* + 1)^{-1}$  to find

$$\mathcal{L} = \int d^4\theta \Phi_0^\dagger \Phi_0 + \left\{ \int d^2\theta \left[ -\frac{1}{8} \Lambda [(1 - Z)^{1/2} \Phi_0]^2 + W(\sqrt{Z} \Phi_0) \right] + \text{h.c.} \right\} \quad (\text{A-8})$$

where

$$1 - Z = \frac{\xi^2}{1 + \xi^2 + \square_*}. \quad (\text{A-9})$$

Eq.(A-8) is an exact result of integrating out the massive field  $\chi$  to *all orders* in  $\partial/\Lambda$ . The global effect is a wavefunction renormalization, albeit in operatorial sense. The presence of  $\square$  inside  $W$  generates (ghost-free) derivative interactions. The derivative acting in the (denominator of the) bilinear F-term brings ghost superfields (when Taylor expanded and truncated, see Section 2). Its structure is *non-local* and follows from the sum of the whole series, as expected from the discussion in Introduction. Note that this term is equivalent to  $(-\Lambda/8) \Phi_0 (1 - Z) \Phi_0$ . In the familiar approximation  $\square_* \ll 1$  one finds  $\mathcal{L}_{\text{eff}}$  of eq.(A-4).

## A.2 “Unfolding” the effective operators to all orders

For further insight into the momentum expansion of an effective theory, let us examine the result of eq.(A-8) by using the “unfolding” procedure. We would like to see if such “unfolded” version still exists to *all orders* in momentum, i.e. we do not “truncate” the Lagrangian after integrating  $\chi$  in (A-1). To this end, we use as many constraints as needed to enforce the solution in eq.(A-3). Introduce

$$\begin{aligned}\overline{D}^2\Phi_0^\dagger &= m\Phi_1 \\ \overline{D}^2\Phi_1^\dagger &= m\Phi_2 \\ &\dots \\ \overline{D}^2\Phi_{n-1}^\dagger &= m\Phi_n, \quad \text{etc.}\end{aligned}\tag{A-10}$$

therefore<sup>17</sup>

$$\chi = -\xi \sum_{j \geq 0} \xi^j \Phi_j, \quad \text{where} \quad \xi \equiv m/\Lambda.\tag{A-11}$$

Define new Lagrange multipliers superfields  $\Sigma_i$   $i=1,2,\dots$ , that enforce constraints (A-10), so we have an equal number of  $\Sigma_i$  and  $\Phi_i$  (with  $i \neq 0$ ). Then integrating  $\chi$  to all orders in momentum gives the Lagrangian below

$$\begin{aligned}\mathcal{L} &= \int d^4\theta \left[ \Phi_0^\dagger \Phi_0 + \xi^2 \left| \sum_{j \geq 0}^n \xi^j \Phi_j \right|^2 \right] + \left\{ \int d^2\theta \frac{1}{8} \Lambda \xi^2 \left( \sum_{j \geq 0}^n \xi^j \Phi_j \right)^2 + \text{h.c.} \right\} \\ &+ \left\{ \int d^2\theta \left[ \frac{-m}{4} \xi \Phi_0 \left( \sum_{j \geq 0}^n \xi^j \Phi_j \right) - \frac{\xi}{4} \sum_{j \geq 1}^n (\overline{D}^2 \Phi_{j-1}^\dagger - m \Phi_j) \Sigma_j + W(\Phi_0) \right] + \text{h.c.} \right\}\end{aligned}\tag{A-12}$$

or equivalently<sup>18</sup>

$$\begin{aligned}\mathcal{L} &= \int d^4\theta \left[ \Phi_0^\dagger \Phi_0 + \xi^2 \left| \sum_{j \geq 0}^n \xi^j \Phi_j \right|^2 + \xi \sum_{j \geq 1}^n (\Phi_{j-1}^\dagger \Sigma_j + \text{h.c.}) \right] \\ &+ \int d^2\theta \left\{ \frac{m\xi}{8} \left[ -\Phi_0^2 + 2 \sum_{j=1}^n \Phi_j \Sigma_j + \left( \sum_{j=1}^n \xi^j \Phi_j \right)^2 \right] + W(\Phi_0) \right\} + \text{h.c.}\end{aligned}\tag{A-13}$$

This is an “unfolding” of Lagrangian (A-7) or (A-8) to all orders in  $(\partial/\Lambda)$  giving the result of

<sup>17</sup>We used in (A-10) the scale  $m$  for dimensional reasons. Using instead the scale  $\Lambda$  does not work since then the eigenvectors of the matrix of kinetic terms would not be normalizable to unity (unitarity violation).

<sup>18</sup> Instead of multiplying the constraint by  $\xi$  one can alternatively demand  $\Sigma \rightarrow 0$  fast enough at large  $\Lambda$ .

integrating out (exactly) the massive state  $\chi$ . Lagrangian (A-13) contains only renormalizable operators by power counting but also an infinite number of superfields<sup>19</sup> ( $n \rightarrow \infty$ ). Truncating the number of superfields to finite  $n$  is equivalent to a truncation of the momentum expansion of the solution  $\chi$  and of the Lagrangian in eq.(A-6). Note the polynomial form of  $\mathcal{L}$ .

Let us diagonalize the hermitian form of the kinetic terms in the first line of  $\mathcal{L}$  in eq.(A-13), in the basis  $(\Phi_0, \Phi_1, \dots, \Phi_n, \Sigma_1, \Sigma_2, \dots, \Sigma_n)$ ,  $n \rightarrow \infty$ . We obtain a squared  $(2n+1) \times (2n+1)$  matrix  $A_n$  ( $n \rightarrow \infty$ ). One can show that this matrix has  $\det A_n = (-1)^n \xi^{4n+2}$ . Since  $\det A_n$  changes sign under  $n \rightarrow n+1$ , each level  $n$  (i.e. new constraint) adds an extra ghost and an extra particle superfields. The characteristic equation is

$$\det(A_n - \lambda I_n) = -(\lambda - \xi)^{n-2}(\lambda + \xi)^{n-2} \mathcal{P}(\lambda, n) = 0 \quad (\text{A-14})$$

where  $n \geq 1$  and

$$\mathcal{P}(\lambda, n) = \lambda^5 + c_4 \lambda^4 + c_3 \lambda^3 + c_2 \lambda^2 + c_1 \lambda + c_0 \quad (\text{A-15})$$

with

$$\begin{aligned} c_4 &= -\frac{1 - \xi^{2n+4}}{1 - \xi^2}, & c_3 &= \xi^2 \left( \frac{1 - \xi^{2n+2}}{1 - \xi^2} - 3 \right), & c_2 &= \xi^2 \left( \frac{1 - \xi^{2n+4}}{1 - \xi^2} + \xi^{2n+2} \right), \\ c_1 &= \xi^4 (1 - \xi^{2n}), & c_0 &= -\xi^{2n+6} \end{aligned}$$

The roots of  $\mathcal{P}(\lambda, n) = 0$  are

$$\begin{aligned} \lambda_0 &= 1 + 2\xi^2 - 2\xi^4 + \mathcal{O}(\xi^5), & \lambda_1 &= \xi^{2n+2} + \mathcal{O}(\xi^{2n+3}); \\ \lambda_2 &= -\xi^2 + 2\xi^4 + \mathcal{O}(\xi^5), & \lambda_{3,4} &= \pm \xi + (1/2)\xi^4 + \mathcal{O}(\xi^5) \end{aligned} \quad (\text{A-16})$$

Of the roots  $\lambda_{0,1,2,3,4}$ , we see that 2 are ghost-like and 3 (or 2) are particle-like for  $n$  finite (infinite) respectively. In addition we have pairs of ghost and particle-like superfields ( $\lambda = \pm \xi$ ) each of degeneracy  $n-2$ , as obvious from the characteristic equation.

Also note  $\mathcal{P}(\lambda, 1)$  contains a factor  $(\lambda^2 - \xi^2)$ , so  $\det(A_1 - \lambda I_1) = -[\lambda^3 - \lambda^2(1 + \xi^2 + \xi^4) - \lambda \xi^2(1 - \xi^2) + \xi^6]$ . The roots are in this case  $\lambda_{0,1,2}$  shown above (with  $n = 1$ ), so we have 2 particle and 1 ghost-like superfields. When truncating  $\mathcal{L}$  of eq.(A-13) to  $n = 1$  and after eliminating  $\Sigma_1$  via its eqs of motion, we obtain exactly eq.(A-4), as expected. Thus eq.(A-13) for  $n = 1$  is a “polynomial” version of eq.(A-4). This case was studied in Section 2 and lead to final eqs.(17), (18) after integrating the (massive) ghosts.

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<sup>19</sup>This is not too surprising. The existence of higher powers of momenta in the expansion (i.e. more derivatives) demands more initial conditions (parameters), in this case extra fields.

The eigenvector corresponding to the eigenvalues  $\lambda_k$ ,  $k$  fixed to  $k = 0, 1, 2, 3, 4$ , is

$$(u^\dagger)_{zk} \equiv N_k (1, \sigma_k \xi, \dots, \sigma_k \xi^{n-1}, \sigma'_k \xi^n; \xi/\lambda_k, \sigma_k \xi^2/\lambda_k, \dots, \sigma_k \xi^n/\lambda_k)^T, \quad (\text{A-17})$$

$$\sigma_k \equiv \frac{\lambda_k^2 - \xi^2 - \lambda_k}{\lambda_k^2 - \xi^2}, \quad \sigma'_k = \frac{\lambda_k^2 - \lambda_0 - \xi^2}{\lambda_k^2}, \quad k = 0, 1, 2, 3, 4; \quad z = 0, 1, 2, \dots, 2n.$$

$N_k$  is a constant of normalization to unity of  $(U^\dagger)_{jk}$  where  $k$  is fixed. With  $|\xi| < 1$  ( $|m| < \Lambda$ ), these eigenvectors ( $k$  fixed) are indeed normalizable for  $n \rightarrow \infty$  ( $N_k < \infty$ ), so unitarity is not violated. Further, for an eigenvalue  $\lambda = \xi$ , for finite  $n$  the eigenvector is  $(0, \Phi_1, \dots, \Phi_{n-1}, 0, 0, \Phi_1, \dots, \Phi_{n-1})^T$  and if  $\lambda = -\xi$ , it is  $(0, \Phi_1, \dots, \Phi_{n-1}, 0, 0, -\Phi_1, \dots, -\Phi_{n-1})^T$ . In both cases  $\lambda = \pm\xi$ , the fields  $\Phi_j$  are arbitrary up to the constraint  $\sum_{j=1}^{n-1} \xi^j \Phi_j = 0$ ; one can choose any two fields to implement this constraint with the remaining fields set to 0.

The Lagrangian in the diagonal basis becomes, after a rescaling  $\tilde{\Phi}_z \rightarrow \tilde{\Phi}_z/\sqrt{|\lambda_z|}$ :

$$\mathcal{L} = \int d^4\theta \left[ \tilde{\Phi}_0^\dagger \tilde{\Phi}_0 + \tilde{\Phi}_j^\dagger \tilde{\Phi}_j - \tilde{\Phi}_{j+n}^\dagger \tilde{\Phi}_{j+n} \right] + \int d^2\theta \left[ U_{zz'} \frac{\tilde{\Phi}_z \tilde{\Phi}_{z'}}{\sqrt{|\lambda_z \lambda_{z'}|}} + W \left( \frac{u_{0z}^\dagger \tilde{\Phi}_z}{\sqrt{|\lambda_z|}} \right) \right] + \text{h.c.}$$

where

$$U_{zz'} = \frac{m\xi}{8} \left[ -u_{0z}^\dagger u_{0z'}^\dagger + 2u_{jz}^\dagger u_{j+n,z'}^\dagger + \xi^j \xi^k u_{jz}^\dagger u_{kz'}^\dagger \right] \quad (\text{A-18})$$

and where sums (not shown) are understood over the repeated indices ( $n$  fixed), with

$$j, k = 1, 2, \dots, n; \quad z, z' = 0, 1, 2, \dots, 2n; \quad \lambda_z = \{ \lambda_0, \lambda_1, \lambda_3, \underbrace{\xi, \dots, \xi}_{n-2}; \lambda_2, \lambda_4, \underbrace{-\xi, \dots, -\xi}_{n-2} \} \quad (\text{A-19})$$

$\tilde{\Phi}_{n+1}, \dots, \tilde{\Phi}_{2n}$  are  $n$  the ghost superfields. From the F-term bilinears and ignoring contributions from  $W$ , it can be shown that one state is light (original particle  $\Phi_0$ ) while the other (particle and ghost-like) superfields are massive (mass of order  $\Lambda$ ).

All superderivatives generated after the integration of massive superfield  $\chi$  were eliminated. The above Lagrangian has only interactions polynomial in superfields and all operators of  $d > 4$  were eliminated via superfield constraints<sup>20</sup>. The downside is the presence of an infinite set of fields, all massive, beyond initial  $\Phi_0$ . These can be integrated out as we did in the “truncated” case. This description is classically equivalent to  $\mathcal{L}$  of eq.(A-1), (A-8) and may be useful in applications. This method can also be applied to more complicated cases.

<sup>20</sup>A similar example exists for the effective Akulov-Volkov action for the goldstino; this action can be completely expressed in terms of superfields, in a interaction-free theory  $L = \int d^4\theta G^\dagger G + \int d^2\theta f G + \text{h.c.}$ ,  $f \neq 0$ , endowed with the constraint  $G^2 = 0$  where  $G$  is the goldstino superfield [42, 43, 44, 45, 46, 47].

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## References

- [1] W. Buchmuller and D. Wyler, “Effective Lagrangian Analysis of New Interactions and Flavor Conservation,” Nucl. Phys. B **268** (1986) 621.
- [2] H. Georgi, “Effective field theory,” Ann. Rev. Nucl. Part. Sci. **43** (1993) 209.
- [3] S. W. Hawking and T. Hertog, “Living with ghosts,” Phys. Rev. D **65** (2002) 103515 [hep-th/0107088].
- [4] S. W. Hawking, “Who’s Afraid Of (higher Derivative) Ghosts?” in “Quantum field theory and quantum statistics,” vol.2, 129, 1985. Eds. A. Batalin et al, Print-86-0124 (Cambridge). Paper written in honor of the 60th birthday of E.S. Fradkin.
- [5] J. F. Oliver, J. Papavassiliou and A. Santamaria, “Can power corrections be reliably computed in models with extra dimensions?,” Phys. Rev. D **67** (2003) 125004 [hep-ph/0302083].
- [6] D. M. Ghilencea, H. M. Lee and K. Schmidt-Hoberg, “Higher derivatives and brane-localised kinetic terms in gauge theories on orbifolds,” JHEP **0608** (2006) 009 [hep-ph/0604215].
- [7] D. M. Ghilencea, “Compact dimensions and their radiative mixing,” Phys. Rev. D **70** (2004) 045018 [hep-ph/0311264].
- [8] D. M. Ghilencea, “Threshold effects near the compactification scale for gauge couplings on orbifolds,” E-print:hep-ph/0612185.
- [9] S. Groot Nibbelink and M. Hillenbach, “Renormalization of supersymmetric gauge theories on orbifolds: Brane gauge couplings and higher derivative operators,” Phys. Lett. B **616** (2005) 125 [hep-th/0503153].
- [10] S. Groot Nibbelink and M. Hillenbach, “Renormalization of supersymmetric gauge theories on orbifolds: Brane gauge couplings and higher derivative operators,” AIP Conf. Proc. **805** (2006) 463.
- [11] S. Groot Nibbelink and M. Hillenbach, “Quantum corrections to non-Abelian SUSY theories on orbifolds,” Nucl. Phys. B **748** (2006) 60 [hep-th/0602155].

- [12] D. M. Ghilencea, “Higher derivative operators as loop counterterms in one-dimensional field theory orbifolds,” *JHEP* **0503** (2005) 009 [hep-ph/0409214].
- [13] D. M. Ghilencea and H. M. Lee, “Higher derivative operators from transmission of supersymmetry breaking on  $S(1) / Z(2)$ ,” *JHEP* **0509** (2005) 024 [hep-ph/0505187].
- [14] A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis and R. Rattazzi, “Causality, analyticity and an IR obstruction to UV completion,” *JHEP* **0610** (2006) 014 [hep-th/0602178].
- [15] I. Antoniadis, E. Dudas and D. M. Ghilencea, “Supersymmetric Models with Higher Dimensional Operators,” *JHEP* **0803** (2008) 045 [arXiv:0708.0383 [hep-th]]. For the non-supersymmetric case see [3, 4].
- [16] I. Antoniadis, E. Dudas, D. M. Ghilencea and P. Tziveloglou, “Higher Dimensional Operators in the MSSM,” *AIP Conf. Proc.* **1078** (2009) 175 [arXiv:0809.4598 [hep-ph]].
- [17] See also Appendix A in I. Antoniadis, E. Dudas, D. M. Ghilencea and P. Tziveloglou, “MSSM with Dimension-five Operators (MSSM(5)),” *Nucl. Phys. B* **808** (2009) 155 [arXiv:0806.3778 [hep-ph]].
- [18] See discussion in Appendix B in: I. Antoniadis, E. Dudas, D. M. Ghilencea and P. Tziveloglou, “MSSM Higgs with dimension-six operators,” *Nucl. Phys. B* **831** (2010) 133 [arXiv:0910.1100 [hep-ph]].
- [19] M. Koehn, J. L. Lehnert and B. A. Ovrut, “Higher-Derivative Chiral Superfield Actions Coupled to  $N=1$  Supergravity,” *Phys. Rev. D* **86** (2012) 085019 [arXiv:1207.3798].
- [20] J. Khoury, J. L. Lehnert and B. Ovrut, “Supersymmetric  $P(X, \phi)$  and the Ghost Condensate,” *Phys. Rev. D* **83** (2011) 125031 [arXiv:1012.3748 [hep-th]].
- [21] M. Koehn, J. L. Lehnert and B. Ovrut, “Ghost condensate in  $N = 1$  supergravity,” *Phys. Rev. D* **87** (2013) 6, 065022 [arXiv:1212.2185 [hep-th]].
- [22] M. Koehn, J. L. Lehnert and B. A. Ovrut, “Scalars with Higher Derivatives in Supergravity and Cosmology,” *Springer Proc. Phys.* **153** (2014) 115.
- [23] R. Bufalo and B. M. Pimentel, “Higher-derivative non-Abelian gauge fields via the Faddeev-Jackiw formalism,” *Eur. Phys. J. C* **74** (2014) 2993 [arXiv:1406.2941 [hep-th]].
- [24] M. Nitta and S. Sasaki, “BPS States in Supersymmetric Chiral Models with Higher Derivative Terms,” *Phys. Rev. D* **90** (2014) 10, 105001 [arXiv:1406.7647 [hep-th]].

- [25] F. S. Gama, M. Gomes, J. R. Nascimento, A. Y. Petrov and A. J. da Silva, “On the one-loop effective potential in the higher-derivative four-dimensional chiral superfield theory with a nonconventional kinetic term,” *Phys. Lett. B* **733** (2014) 247 [arXiv:1401.5414 [hep-th]].
- [26] F. S. Gama, J. R. Nascimento and A. Y. Petrov, “Effective superpotential in the generic higher-derivative superfield supersymmetric three-dimensional gauge theory,” *Phys. Rev. D* **88** (2013) 4, 045021 [arXiv:1307.3190 [hep-th]].
- [27] F. S. Gama, J. R. Nascimento and A. Y. Petrov, “Effective superpotential in the generic higher-derivative three-dimensional scalar superfield theory,” *Phys. Rev. D* **88** (2013) 6, 065029 [arXiv:1308.5834 [hep-th]].
- [28] E. A. Gallegos, C. R. Senise, Jr. and A. J. da Silva, “Higher-Derivative Wess-Zumino Model in Three Dimensions,” *Phys. Rev. D* **87** (2013) 085032 [arXiv:1212.6613 [hep-th]].
- [29] M. Carena, K. Kong, E. Ponton and J. Zurita, “Supersymmetric Higgs Bosons and Beyond,” *Phys. Rev. D* **81** (2010) 015001 [arXiv:0909.5434 [hep-ph]].
- [30] R. Flauger, S. Hellerman, C. Schmidt-Colinet and M. Sudano, “The One-Loop Effective Kähler Potential. I: Chiral Multiplets,” arXiv:1205.3492 [hep-th].
- [31] W. Buchmuller and S. T. Love, “Chiral Symmetry and Supersymmetry in the Nambu-Jona-Lasinio Model,” *Nucl. Phys. B* **204** (1982) 213; W. Buchmuller and U. Ellwanger, “On the Structure of Composite Goldstone Supermultiplets,” *Nucl. Phys. B* **245** (1984) 237.
- [32] S. Ferrara, J. Wess and B. Zumino, “Supergauge Multiplets and Superfields,” *Phys. Lett. B* **51** (1974) 239; S. Cecotti, S. Ferrara and M. Villasante, “Linear Multiplets and Super Chern-Simons Forms in 4D Supergravity,” *Int. J. Mod. Phys. A* **2** (1987) 1839; P. Binetrui, G. Girardi and R. Grimm, “Linear supermultiplets and nonholomorphic gauge coupling functions,” *Phys. Lett. B* **265** (1991) 111.
- [33] See e.g. S. Ferrara, R. Kallosh, A. Linde and M. Porrati, “Minimal Supergravity Models of Inflation,” *Phys. Rev. D* **88** (2013) 8, 085038 [arXiv:1307.7696 [hep-th]].
- [34] P. J. Fox, A. E. Nelson and N. Weiner, “Dirac gaugino masses and supersoft supersymmetry breaking,” *JHEP* **0208** (2002) 035 [hep-ph/0206096].
- [35] A. E. Nelson, N. Rius, V. Sanz and M. Unsal, “The Minimal supersymmetric model without a  $\mu$  term,” *JHEP* **0208** (2002) 039 [hep-ph/0206102].

- [36] S. Cecotti, S. Ferrara and L. Girardello, “Structure of the Scalar Potential in General  $N = 1$  Higher Derivative Supergravity in Four-dimensions,” *Phys. Lett. B* **187** (1987) 321; F. Farakos, S. Ferrara, A. Kehagias and M. Porrati, “Supersymmetry Breaking by Higher Dimension Operators,” *Nucl. Phys. B* **879** (2014) 348 [arXiv:1309.1476 [hep-th]].
- [37] C. P. Burgess, J. -P. Derendinger, F. Quevedo and M. Quiros, “Gaugino condensates and chiral linear duality: An Effective Lagrangian analysis,” *Phys. Lett. B* **348** (1995) 428 [hep-th/9501065] and *Annals Phys.* **250** (1996) 193 [hep-th/9505171];
- [38] P. Binetrui, M. K. Gaillard and T. R. Taylor, “Dynamical supersymmetric breaking and the linear multiplet,” *Nucl. Phys. B* **455** (1995) 97 [hep-th/9504143].
- [39] S. J. Gates, Jr., “Super P Form Gauge Superfields,” *Nucl. Phys. B* **184** (1981) 381.
- [40] N. Kaloper, A. Lawrence and L. Sorbo, “An Ignoble Approach to Large Field Inflation,” *JCAP* **1103** (2011) 023 [arXiv:1101.0026 [hep-th]]; K. Groh, J. Louis and J. Sommerfeld, “Duality and Couplings of 3-Form-Multiplets in  $N=1$  Supersymmetry,” *JHEP* **1305** (2013) 001 [arXiv:1212.4639 [hep-th]]; E. Dudas, “Three-form multiplet and Inflation,” *JHEP* **1412** (2014) 014 [arXiv:1407.5688 [hep-th]].
- [41] J. Wess and J. Bagger, “Supersymmetry and supergravity”, Princeton University Press, 1992, 2nd edition.
- [42] M. Rocek, “Linearizing the Volkov-Akulov Model,” *Phys. Rev. Lett.* **41** (1978) 451.
- [43] U. Lindstrom, M. Rocek, “Constrained Local Superfields”, *Phys. Rev. D* **19** (1979) 2300.
- [44] E. A. Ivanov and A. A. Kapustnikov, “General Relationship Between Linear and Non-linear Realizations of Supersymmetry”, *J. Phys. A* **11** (1978) 2375.
- [45] R. Casalbuoni, S. De Curtis, D. Dominici, F. Feruglio and R. Gatto, “Nonlinear Realization of Supersymmetry Algebra From Supersymmetric Constraint,” *Phys. Lett. B* **220** (1989) 569.
- [46] A. Brignole, F. Feruglio and F. Zwirner, “On the effective interactions of a light gravitino with matter fermions,” *JHEP* **9711** (1997) 001 [hep-th/9709111].
- [47] Z. Komargodski and N. Seiberg, “From Linear SUSY to Constrained Superfields,” *JHEP* **0909** (2009) 066 [arXiv:0907.2441 [hep-th]].